Polynomials

Resultants

Resultant of Two Polynomials

$$Q(X) = AX^2 + 2BX + C$$

$$R(X) = DX^2 + 2EX + F$$

$$\mathbf{R}(Q,R) = f(A,B,C,D,E,F)$$

$$\mathbf{R} = 0 \quad \text{Û} \quad Q \text{ and } R \text{ have a common root}$$

Calculating the Resultant

$$Q(X) = AX^2 + 2BX + C = 0$$

$$R(X) = DX^2 + 2EX + F = 0$$

$$aQ + bR = 0$$

$$DQ - AR = 0$$

$$D\left(AX^2 + 2BX + C\right)$$

$$-A\left(DX^2 + 2EX + F\right) = 0$$

$$2(BD - AE)X + (CD - AF) = 0$$

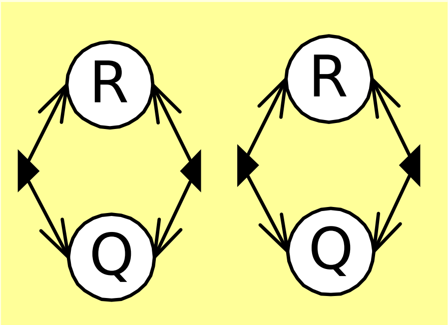
Resultant of Q and R

$$Q(x,w) = Ax^2 + 2Bxw + Cw^2$$

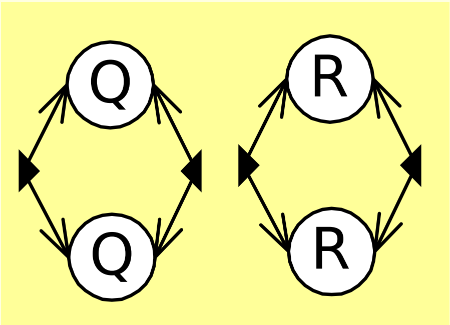
$$R(x,w) = Dx^2 + 2Exw + Fw^2$$

$$\mathbf{R}(Q,R) = +A^2F^2 - 4ABEF + 4ACE^2 - 2ACDF$$
$$+4B^2DF - 4BCED + C^2D^2$$

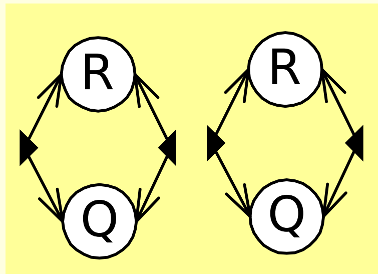
$\mathbf{R}(Q,R) =$

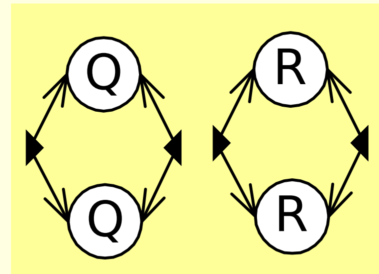


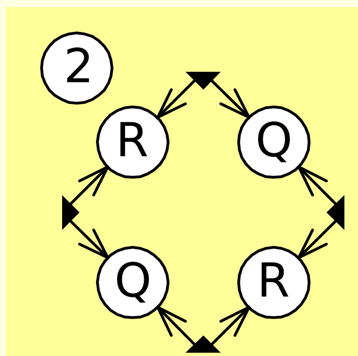
-

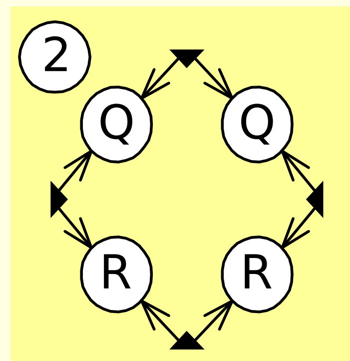


Two Forms of Resultant

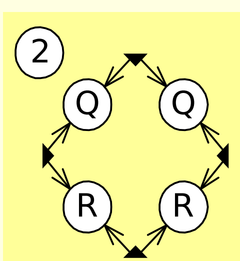
$\mathbf{R}(Q,R) =$ 

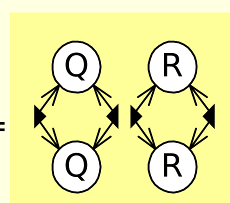
 - 

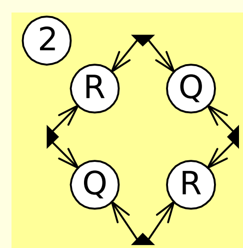
$\mathbf{R}(Q,R) =$ 

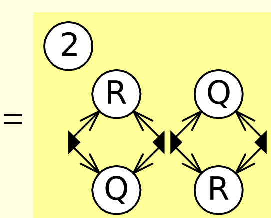
 - 

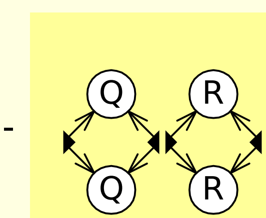
Identities:



= 



= 

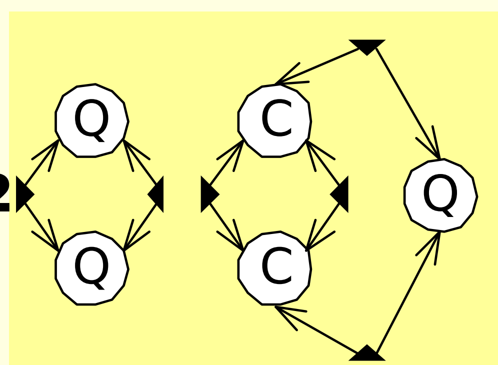
 - 

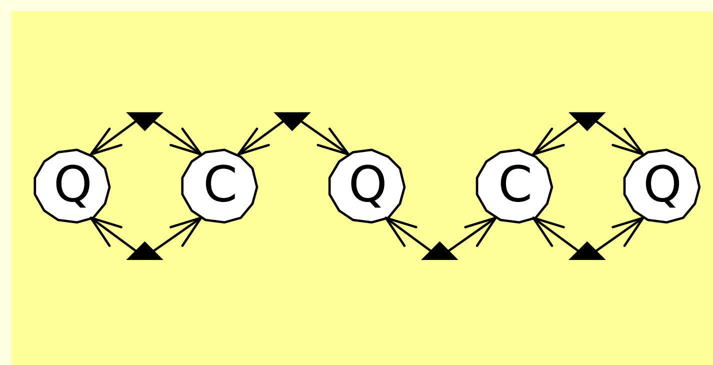
Resultant of Q and C

$$\mathbf{C}(x,w) = Ax^3 + 3Bx^2w + 3Cxw^2 + Dw^3$$

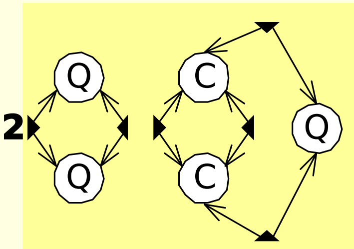
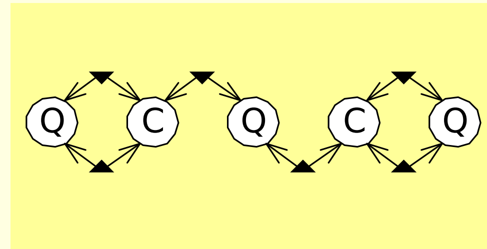
$$\mathbf{Q}(x,w) = Ex^2 + 2Fxw + Gw^2$$

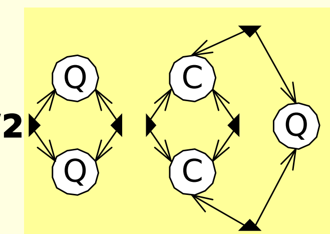
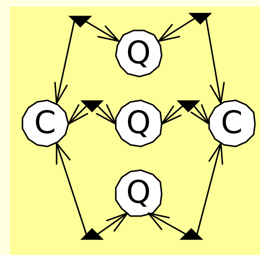
$$\begin{aligned}\mathbf{R}(\mathbf{Q},\mathbf{C}) = & -A^2\left(G^3\right) + 6AB\left(FG^2\right) - 6AC\left(2F^2G - EG^2\right) - 2AD\left(-4F^3 + 3EFG\right) \\ & - 9B^2\left(EG^2\right) + 18BC\left(EFG\right) - 6BD\left(2EF^2 - E^2G\right) \\ & - 9C^2\left(E^2G\right) + 6CD\left(E^2F\right) \\ & - D^2\left(E^3\right)\end{aligned}$$

$\mathbf{R}(\mathbf{Q},\mathbf{C}) = -\mathbf{2}$ 

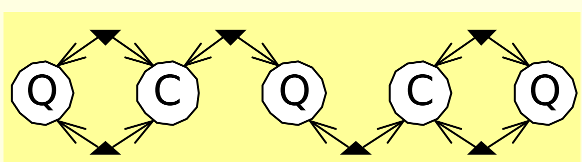
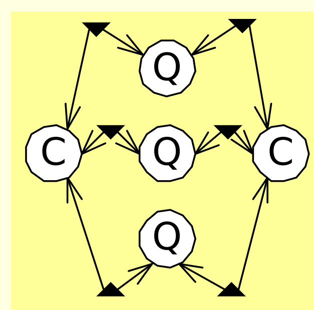
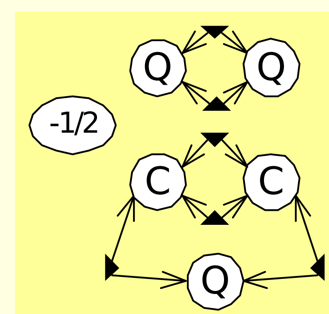
$-$ 

Forms of Q,C Resultant

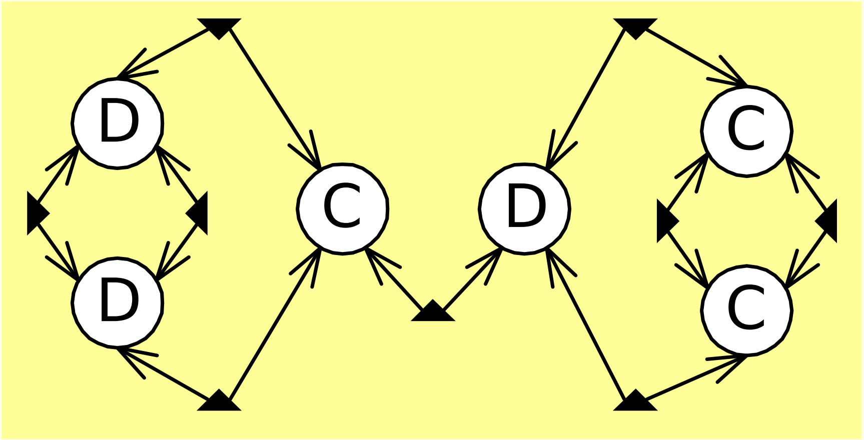
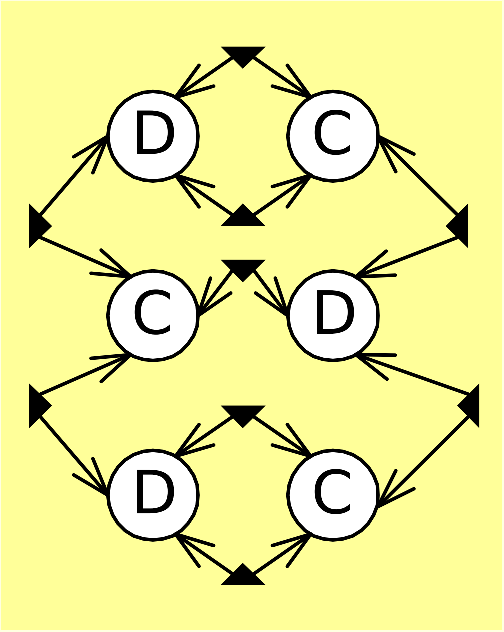
R(Q,C) = - 2  - 

R(Q,C) = - 3/2  + 

Identity:

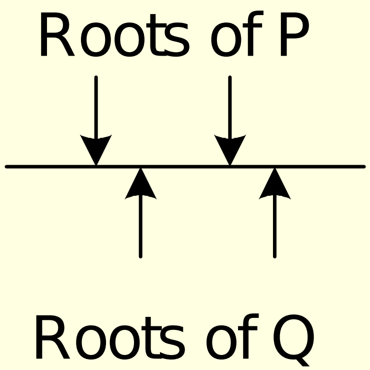
 = -  + 

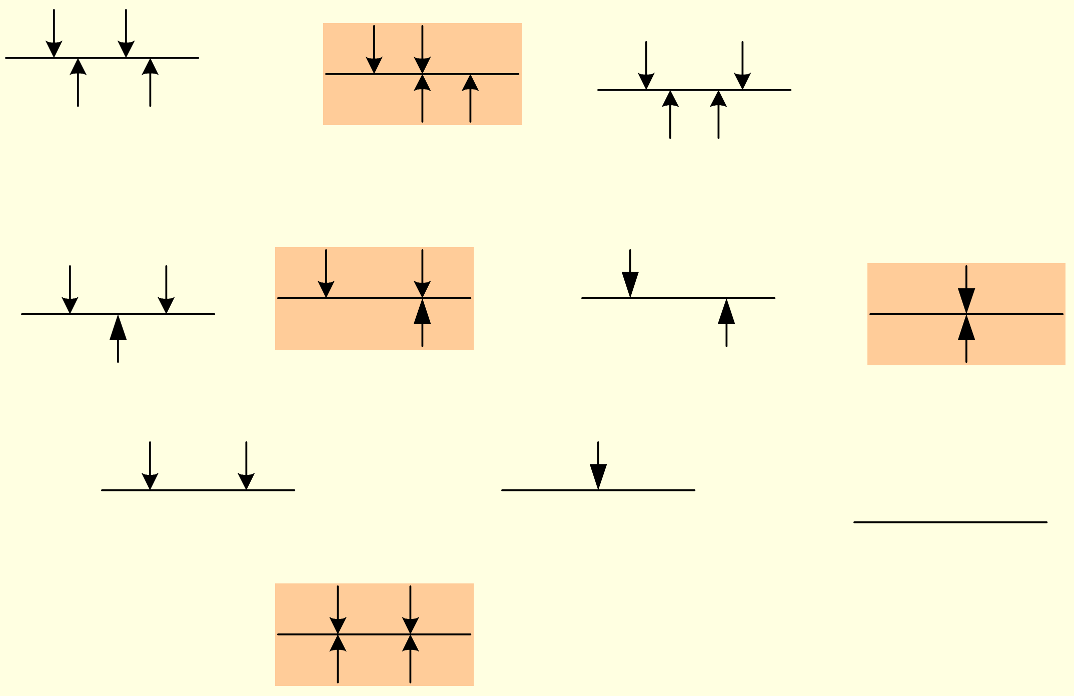
Resultant of two Cubics ?

2  - 

What’s Really Going on With Resultants?

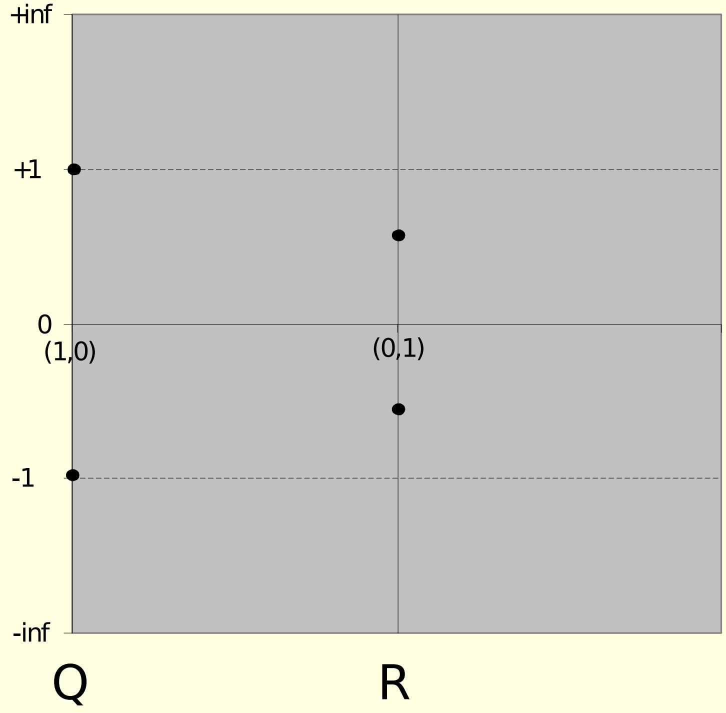
Possible Relationships between P and Q





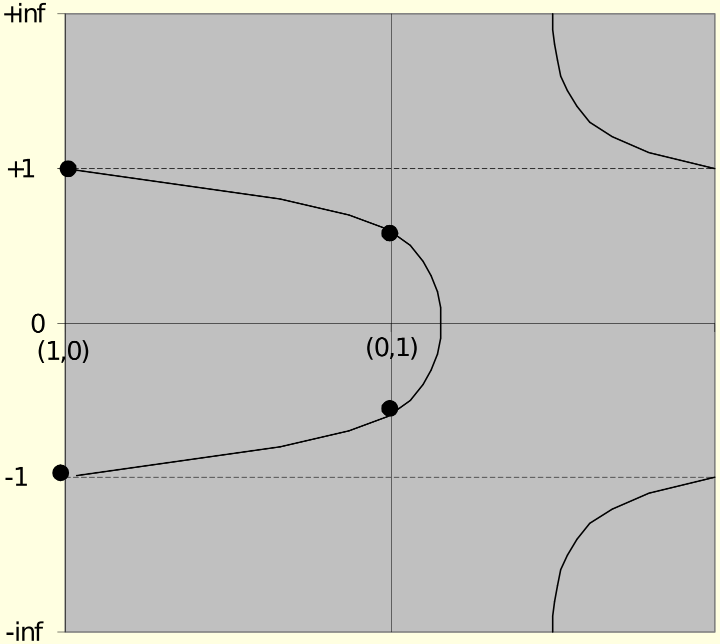
Linear Combos of Q and R

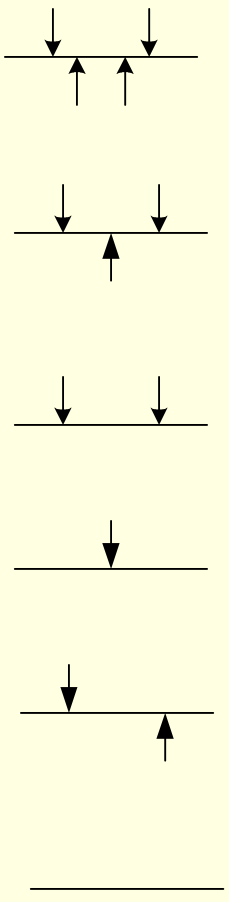
$$aQ + bR = 0$$



Q,R Have Enclosed Roots

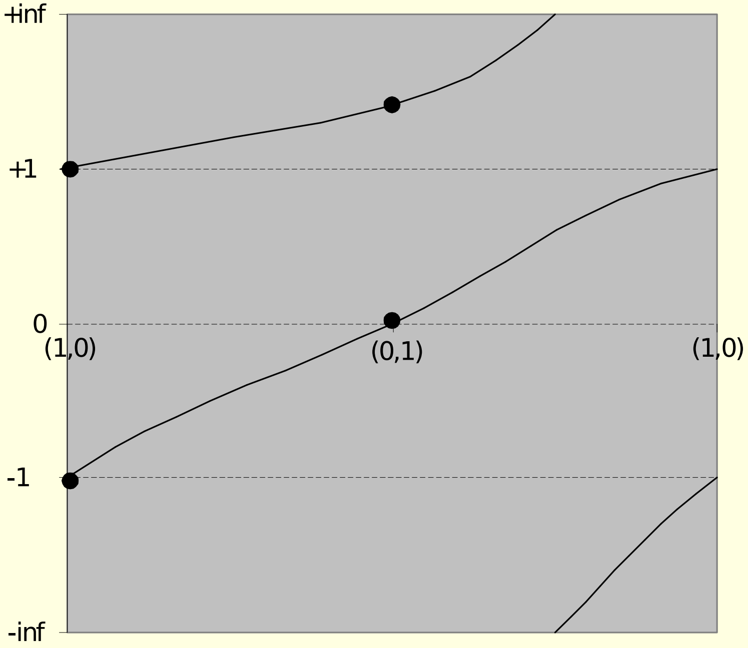
$$aQ + bR = 0$$

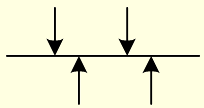




Q,R Have Interleaved Roots

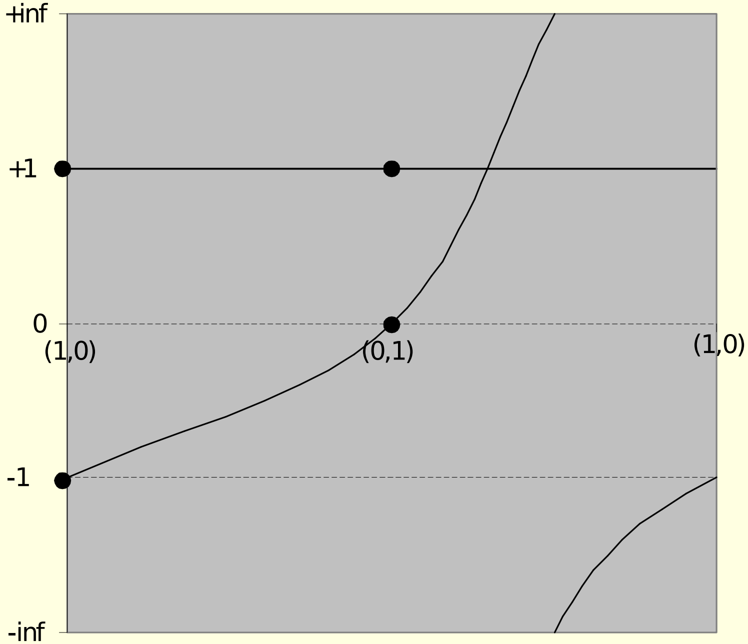
$$aQ + bR = 0$$

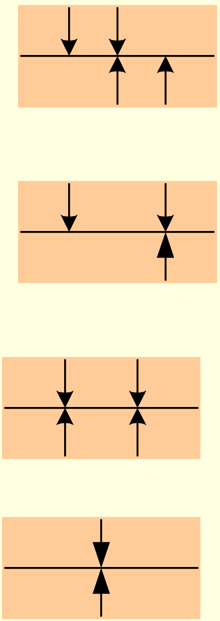




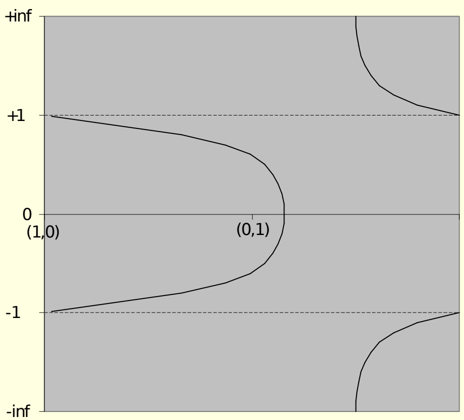
Q,R Have Common Root

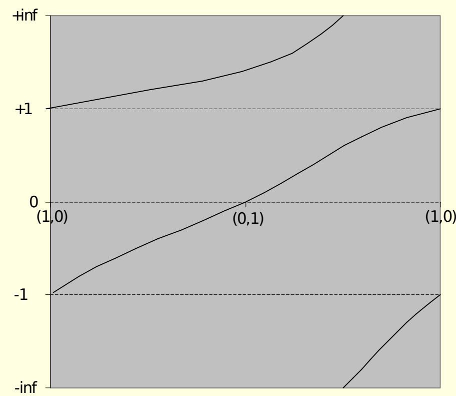
$$aQ + bR = 0$$

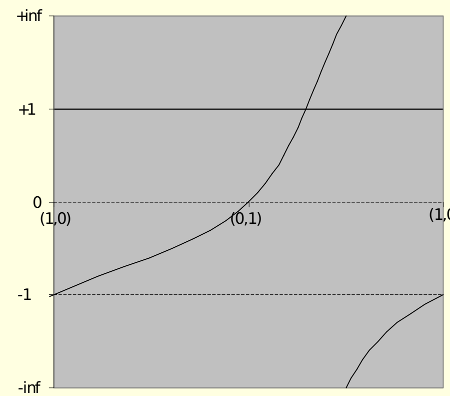


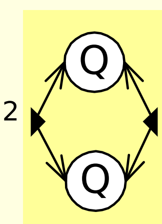
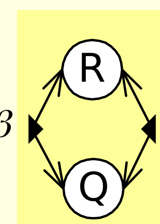
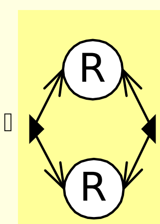


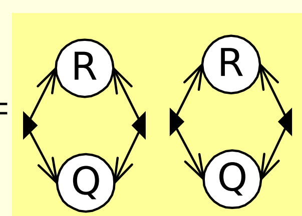
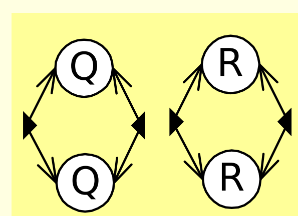
Three Possible Evolutions of Roots of $aQ+bR$







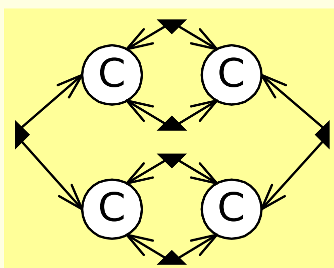
$$\det\left(aQ+bR\right) = \alpha^{2}$$  $+2\alpha\beta$  $+\beta^{2}$ 

$$\mathbf{R}(Q,R) =$$  - 

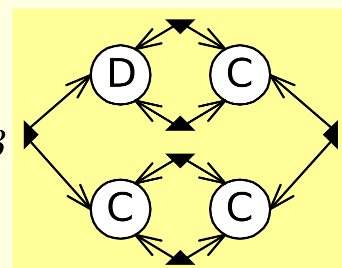
Possible Evolutions of Roots of Two Cubics

$$\det\left(a\mathbf{C}+b\mathbf{D}\right)=$$

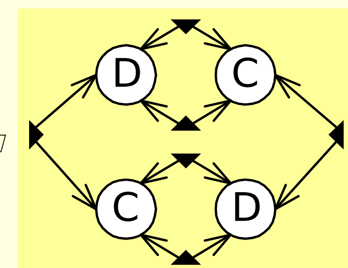
α^4



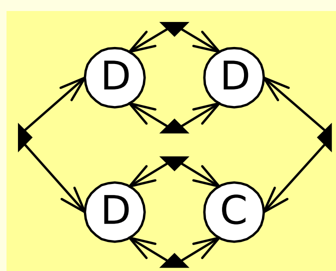
$+4\alpha^3\beta$



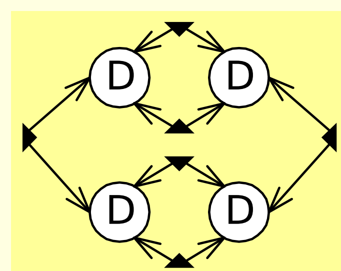
$+\square\,\alpha^{\square}\beta^{\square}$



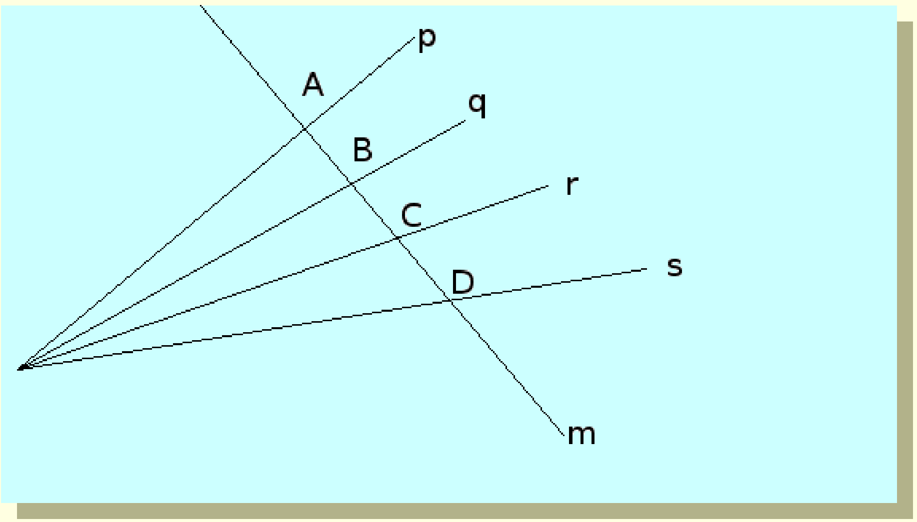
$+\square\,\alpha\beta^{\square}$



$+\beta^{\square}$



The Cross Ratio



$$\chi = \frac{|\mathbf{AB}|/|\mathbf{BD}|}{|\mathbf{AC}|/|\mathbf{CD}|}$$

$$c = \frac{(\mathbf{p}\times\mathbf{q}\cdot\mathbf{m})(\mathbf{r}\times\mathbf{s}\cdot\mathbf{m})}{(\mathbf{q}\times\mathbf{s}\cdot\mathbf{m})(\mathbf{p}\times\mathbf{r}\cdot\mathbf{m})}$$

Generalized Cross Ratio of Two Quadratic Polynomials

$\chi =$